Chapter 10

Confidence Intervals

When we select a sample, we want to infer some conclusion about the population that the sample represents. In this chapter, we will be introduced to one type of formal statistical inference: Confidence Intervals. It is based on the sampling distributions of statistics. In this chapter, we concentrate on the reasoning of inference. We will start by pretending the world is simpler than it is. Then, we will become more realistic and make predictions that are more in line with practical uses.

Objectives:

- Describe Statistical Inference
- Describe the basic form of all confidence intervals.
- Construct and interpret a confidence interval for a population mean (including paired data) and for a population proportion.
- Describe a margin of error and explain ways in which you can control the size of the margin of error.
- Determine the sample size necessary to construct a confidence interval for a fixed margin of error.
- Compare and contrast the t distribution and the normal distribution.
- List the conditions that must be present to construct a confidence level for a population mean or a population proportion.
- Explain what is meant by the standard error, and determine the standard error of $\overline{x}$ and the standard error of $\hat{p}$.

CASE CLOSED:

Need help? Give us a call!

If your cable television goes out, you phone the cable company to get it fixed. Does a real person answer your call? These days, probably not. It is far more likely that you will get an automated response. You will probably be offered several options, such as: to order cable service, press 1; for questions about your bill, press 2; to add new channels, press 3; (and finally) to speak with a customer service agent, press 4. Callers will get frustrated if they have to wait too long before speaking to a live voice. So companies try hard to minimize the time required to connect to a customer service representative.

A large bank decided to study the call response times in its customer service department. The bank's goal was to have a representative answer an incoming call within 30 seconds. Here are the results from a random sample of 300 calls to the bank's customer service center.
Table 10.1 shows the time from the first ring until a customer service agent answers. By the end of this chapter, you will learn how to use these sample data to provide estimates of how well the bank is doing in meeting its call response goal.

Table 10.1 *Call center response times (seconds)*

59	13	2	24	11	18	38	12	46	17	77	12	46
44	4	74	41	22	25	7	10	46	78	14	6	9
122	8	16	15	17	17	9	15	24	70	9	9	10
32	9	68	8	10	41	13	17	50	12	82	97	33
76	56	42	19	14	21	12	44	63	5	21	11	47
8	12	4	111	37	12	24	43	37	27	65	32	3
9	26	5	10	30	27	21	14	19	44	49	10	24
11	10	22	43	70	27	10	32	96	11	29	7	28
22	17	9	24	15	14	34	5	38	29	16	65	6
5	58	17	7	44	14	16	4	46	32	52	75	11
11	17	31	8	36	25	14	85	4	46	23	58	5
54	28	6	46	4	28	11	111	6	3	83	27	6
83	27	2	56	26	21	276	14	30	8	7	12	4
29	21	23	4	14	23	22	19	66	51	60	14	111
20	7	7	87	22	11	53	20	14	41	30	7	10
11	9	9	101	55	18	20	77	14	13	11	22	15
2	14	20	83	25	10	34	23	21	5	14	22	10
68	8	70	56	8	26	7	15	7	9	144	11	109
20	4	16	20	124	16	16	47	97	27	61	35	18
22	244	19	10	6	43	20	77	22	7	33	67	20
4	28	5	7	118	18	1	35	78	35	71	85	24
333	50	11	12	11	13	19	16	91	4	63	14	22
43	25	18	55	13	11	6	13	4	3	17	11	6
17												

Source: Haipeng Shen, "Nonparametric regression for problems involving lognormal distributions," PhD thesis, University of Pennsylvania, 2003. Thanks to Haipeng Shen and Larry Brown for sharing the data.

Statistical Inference:

Statistical Inference provides methods for drawing conclusions about a population from sample data.

What does confidence interval mean?

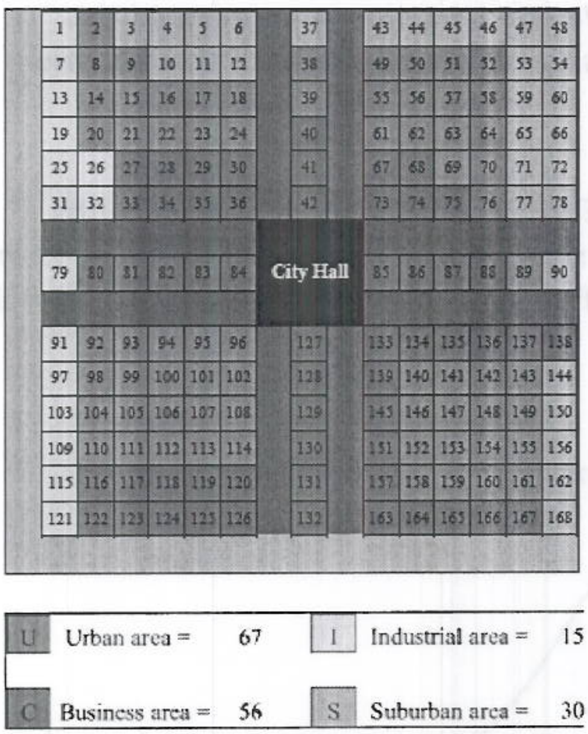
The Town of Newton sits next to two rivers, the Saco and Vanzetti. Newton is made up of 168 land parcels, all the same size. The town is shown to the right. The colors refer to the use of the land and are shown in the legend below. There is not enough money for the town to run a full census. It is an expensive process and many people need to be hired. It is decided to estimate the population of the town by sampling techniques. It is thought that a sample of 21 parcels of land will be sampled which is one-eighth of the actual number of parcels.
So an SRS of 21 samples will be done: Using the simulation, let's choose an SRS. Calculate the sample mean and standard deviation for your sample.

Pull one sample for all of us to use.

Sample mean: $\bar{x}$= _________ Sample St. Dev: s =____________

$\bar{x}$ then make a prediction of pop

1



Industrial: (15)
1,7,13,19,25,26,31,32,79,91,97,103,109,115,121

56
Business: 27- 30, 33-34, 41-42, 67-68, 73-75, 80-88, 116-120, 122-139, 145, 151, 157, 163-167

(67)
Urban: 2, 8-9, 14-18, 20-24, 38-39, 49-52, 55-58, 61-64, 69-70, 76,89, 92-,96, 98-102,104-108, 110-114, 140-143,146-149,152-155, 158-161

(30)
Suburban: 3-6, 10-12, 37, 43-48, 53-54, 59-60, 65-66, 71-72,77-78,90,144,150,156,162,168

Real pop:

We are interested in μ, the unknown population mean of the 168 parcels. If we can find that, we can multiply it by 168 and we will have our population.

The thought process is that if $\bar{x}$ = _______, then μ = _________ as our sample mean $\bar{x}$ is an unbiased estimator of our population mean μ. We expect μ to be approximately the value of $\bar{x}$. However, if we took another sample of 21 land parcels, we would get a different $\bar{x}$ and thus a different estimation of μ.

point out sampling variability

Recall what the Central Limit Theorem tells us about sampling distributions:

1)$\overline{x}$ has a normal distribution. If we were to take many samples of 21 parcels, the result would be bell-shaped and follow our 68-95-99.7% rule.

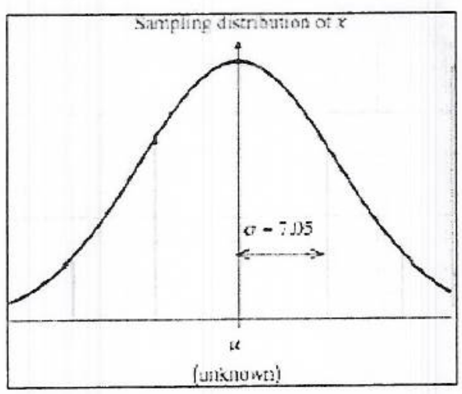
2) The mean of our normal sampling distribution is the same as our unknown population mean μ

3) The standard deviation of $\overline{x}$ for our 21 parcels is $\frac{\sigma}{\sqrt{21}}$ where σ is the standard deviation of all 168 land parcels.

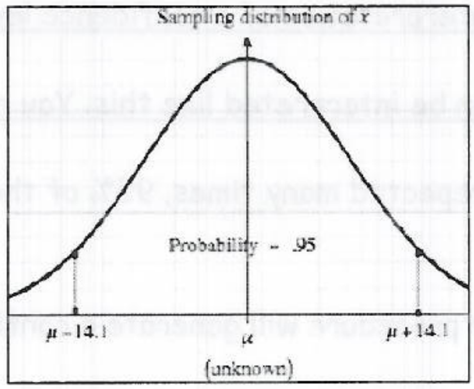
Let us suppose that the standard deviation of population parcels is σ = 32.3. Then the standard deviation of $\overline{x}$ is then 7.05.

Important: Note that this supposition of knowing σ is completely unrealistic. If we knew σ then we'd know μ and this process would be unnecessary. We are doing this to understand the process of hypothesis testing and confidence intervals. Later we will deal with the realistic situation that we don't know the value of σ.

The picture below shows the sampling distribution of $\overline{x}$ in repeated samples of size 21, the sample mean $\overline{x}$ would vary according to the normal distribution with mean equal to the unknown μ and standard deviation is 7.05. The different values of $\overline{x}$ appear along the axis and the normal curve shows how probable these values are.



The picture below is another picture of the sampling distribution. The 68-95-99.7% rules says that in 95% of all samples, $\overline{x}$ will be within two standard deviations of the unknown population mean μ. That is, $\overline{x}$ will be within 14.1 of μ in 95% of all samples. So in 95% of all samples, the unknown μ lies between $\overline{x}$ + 14.1 and $\overline{x}$ - 14.1.



Our sample gave $\bar{x}$ = ________. We say that we are 95% confident that the unknown parcel mean lies between $\bar{x}$ - 14.1 = __________ and $\bar{x}$ + 14.1 = __________. The grounds for our confidence lie in the fact that only one of the two possible facts is true:

1) our true μ lies between our $\bar{x}$ - 14.1 and $\bar{x}$+ 14.1. 95% of all our samples should do so.

2) the true μ does not lie between our $\bar{x}$ - 14.1 and $\bar{x}$ + 14.1. Only 5% of our samples should do so.

VERY IMPORTANT: Remember, you are basing this on one value of $\bar{x}$. We do not know whether our sample is one of the 95% for which the interval $\bar{x}$ ±14.1 "catches" μ or one of the unlucky 5%. The statement that we are 95% confident that the unknown μ lies between _______ and _______ is a quick way of saying: we got these numbers by a procedure that gives correct results 95% of the time. The interval of numbers between $\bar{x}$ ±14.1 is called a 95% confidence interval.

It has the form: Confidence Interval = Estimate ± margin of error

The estimate $\bar{x}$ is our guess for the unknown parameter μ. The margin of error ±14.1 shows how accurate we believe our guess is, based on the variability of the estimate. This is a 95% confidence interval because it catches the unknown μ in 95% of all possible samples.

Interpretation of a confidence level:

So a 95% confidence interval can be interpreted like this: You need to know and understand the wording.

- If the procedure were repeated many times, 95% of the confidence intervals would catch the true mean μ.

- The probability that our procedure will generate a confidence interval containing the true mean μ is 95%.
- We are 95% confident that our true mean μ lies in our interval.

It does Not say:

- ✓ 95% of the population lies in the confidence interval.

- ✓ The probability that the true mean μ is in our confidence interval is 95%.

Again, remember, you will never do this sampling process 100 times or even 10 times. You will do it exactly once. You will be making a judgment based on that one sample. The process will generate an interval that will "catch" the true mean μ 95% of the time.

Example 1: A poll interviewed 1,500 men randomly selected from the United States and found that 64% watched at least one NFL football game a week. The poll announced a ± 3 percentage points margin of error for 95% confidence in its conclusions.

a) What is the 95% confidence interval for American men watching NFL games?

64 ± 3

61% - 67%

b) Explain in correct wording what this says.

We are 95% confident that thru percentage of men who watch at least one NFL game per week is between 61% and 67%

Example 2: A gas station looks at an SRS of 150 customers over a week and finds that the average number of gallons a customer pumps is 8.4 with a margin of error of 2.15 gallons for 95% confidence.

a) What is the 95% confidence interval for customer purchasing gas at this station?

8.4 ± 2.15

6.25 - 10.55 gallons

b) Explain in correct wording what this says.

We are 95% confident that the true average number of gallons pumped per week is between 6.25 and 10.55 gallons

Example 3: The price of a two-liter bottle of Pepsi nationwide has a standard deviation σ of 23.4 cents. A random sample of 50 bottles found an average price of $1.64.

a) What is the 95% confidence interval for the price of a two-liter bottle of Pepsi.

by CLT, $\bar{x}$ is approx Normal since more than 500 bottles of pepsi are produced, $\sigma_{\bar{x}} = 23.4/\sqrt{50} = 3.31$

$1.64 \pm 2(0.0331)$

$(1.57, 1.71)$

b) Explain in correct working what this says.

We are 95% confident that the true average price of a 2-liter of Pepsi is between $1.57 and $1.71

Confidence intervals do not have to be 95% although that is usually what is given in many statistical problems. Other confidence intervals are 90% and 99%. 90% confidence intervals are less confident than 95% while 99% confidence intervals are more confident. We use 95% unless otherwise stated.

For instance, in example 2 above, we found the average number gallons that a customer pumps is 8.4 with a margin of error of 2.15 for 95% confidence. The following table shows a possible margin of error and confidence interval for not only 95% but 90% and 99% as well.

Confidence	$\bar{x}$	Margin of error	Lower bound	Upper bound
90%	8.4	1.4	7.0	9.8
95%	8.4	2.15	6.25	10.55
99%	8.4	2.6	5.8	11.0

Notice as confidence increases, interval becomes wider

We can have any percentage confidence levels although 90%, 95%, and 99% are the most common.

Confidence Interval:

A level *C* confidence interval for a parameter has 2 parts:

1. A confidence level calculated from the data, usually of the form
   Estimate ± margin of error

2. A confidence level *C*, which gives the probability that the interval will capture the true parameter value in repeated values. That is the confidence level is the success rate for the method.

Thurs 3/1

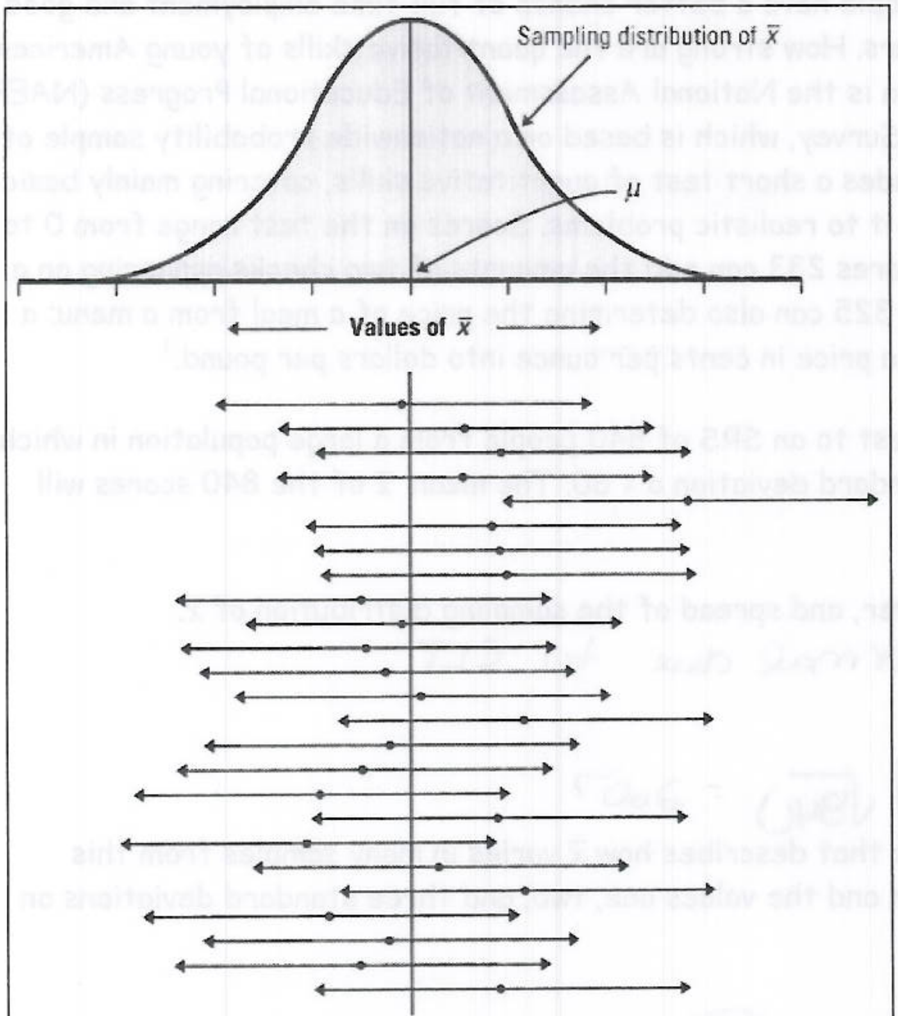


Figure 10.4 *Twenty-five samples from the same population gave these 95% confidence intervals. In the long run, 95% of all samples give an interval that contains the population mean μ.*

The figure shows the result of drawing many SRSs from the same population and calculating a 95% confidence interval from each sample. The center of each interval is at x and therefore varies from sample to sample. The sampling distribution of x appears at the top of the figure to show the long-term pattern of this variation. The 95% confidence intervals from 25 SRSs appear below.

Here's what you should notice:

- The center $\overline{x}$ of each interval is marked by a dot.
- The arrows on either side of the dot span the confidence interval. The distance from the dot to the end of an arrow is the margin of error for that interval.
- 24 of these 25 intervals (that's 96%) cover the true value of μ. If we took all possible samples, 95% of the resulting confidence intervals would contain μ.

Notice that $\overline{x}$ is center of the CI

Example 4: **NAEP scores** Young people have a better chance of full-time employment and good wages if they are good with numbers. How strong are the quantitative skills of young Americans of working age? One source of data is the National Assessment of Educational Progress (NAEP) Young Adult Literacy Assessment Survey, which is based on a nationwide probability sample of households. The NAEP survey includes a short test of quantitative skills, covering mainly basic arithmetic and the ability to apply it to realistic problems. Scores on the test range from 0 to 500. For example, a person who scores 233 can add the amounts of two checks appearing on a bank deposit slip; someone scoring 325 can also determine the price of a meal from a menu; a person scoring 375 can transform a price in cents per ounce into dollars per pound.[1]

Suppose that you give the NAEP test to an SRS of 840 people from a large population in which the scores have mean 280 and standard deviation σ = 60. The mean $\bar{x}$ of the 840 scores will vary if you take repeated samples.

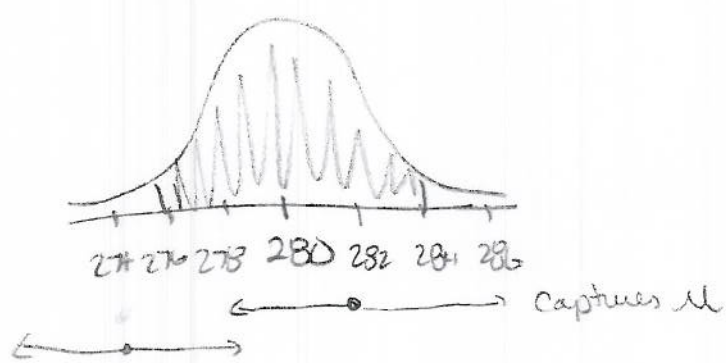
(a) Describe the shape, center, and spread of the sampling distribution of $\bar{x}$.

approx Normal due to CLT

$\mu_{\bar{x}} = 280$

$\sigma_{\bar{x}} = 60 / \sqrt{840} = 2.07$

(b) Sketch the Normal curve that describes how $\bar{x}$ varies in many samples from this population. Mark its mean and the values one, two, and three standard deviations on either side of the mean.



does not capture μ

(c) According to the 68-95-99.7 rule, about 95% of all the values of $\bar{x}$. fall within ______ of the mean of this curve. What is the missing number? Call it *m* for "margin of error." Shade the region from the mean minus *m* to the mean plus *m* on the axis of your sketch.

2 standard deviation

(d) Whenever $\overline{x}$ falls in the region you shaded, the true value of the population mean, $\mu = 280$, lies in the confidence interval between $\overline{x} - m$ and $\overline{x} + m$. Draw the confidence interval below your sketch for one value of $\overline{x}$ inside the shaded region and one value of $\overline{x}$ outside the shaded region.

(e) In what percent of all samples will the true mean $\mu = 280$ be covered by the confidence interval $\overline{x} \pm m$?

95%

Confidence Intervals for a population mean when σ is known

Conditions for Constructing a Confidence Interval for μ:

The construction of a CI for μ is appropriate when:

1. The data comes from a SRS from the population of interest.
2. The sampling distribution of $\overline{x}$ is approximately Normal.
3. Individual observations are independent when sampling without replacement (i.e. $N \geq 10n$)

Be sure to check that these conditions are satisfied before constructing the confidence interval! Show your work every time!

Finding z*: To construct an 80% confidence interval, we must catch the central 80% of the Normal sampling distribution of $\bar{x}$. In catching the central 80% we leave out 20%, or 10% in each tail. So z^* is the point with area 0.1 to its right (and 0.9 to its left) under the standard Normal curve. Search the body of the z-table or use invNorm to find the point with area 0.9 to its left. The closest entry is z^* = 1.28. There is area 0.8 under the standard Normal curve between −1.28 and 1.28. The figure below shows how z^* is related to areas under the curve.

z	.07	.08	.09
1.1	.8790	.8810	.8830
1.2	.8980	.8997	.9015
1.3	.9147	.9162	.9177

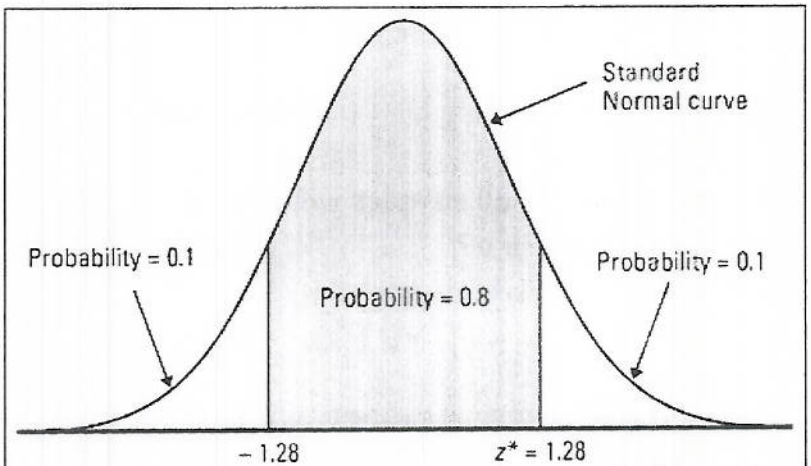


Figure 10.5 The central probability 0.8 under a standard Normal curve lies between −1.28 and 1.28. That is, there is area 0.1 to the right of 1.28 under the curve.

The figure below shows the general situation for any confidence level *C*. If we catch the central area *C*, the leftover tail area is 1 − *C*, or (1 − *C*)/2 for each of the *upper* and *lower tails*. You can find z^* for any *C* by searching Table A. Here are the results for the most common confidence levels:

Confidence level	Tail area	z^*
90%	0.05	1.645
95%	0.025	1.960
99%	0.005	2.576

Critical Values:

The z* Value that determines the central probability C under the Standard Normal Curve.

Finding the Confidence Interval:

1. Under any Normal cure the area between z* deviations above and below its mean is *C*.

2. $\sigma_{x\text{-bar}} = \sigma/\sqrt{n}$ and $\mu_{x\text{-bar}} = \mu$ so there is the probability *C* that the observed value $\bar{x}$ takes a value between $\mu \pm z^* \sigma/\sqrt{n}$.

3. Whenever this happens, the population μ is contained between $\bar{x} \pm z^* \sigma/\sqrt{n}$.

Confidence Interval for a Population Mean (σ known)

Choose of SRS of size n from a population having unknown mean μ and known standard deviation σ. A **level *C* confidence interval** for μ is

$$\bar{x} \pm z^* \sigma/\sqrt{n}$$

Here z* is the value that determines area *C* between -z* and z* under the standard Normal curve. The interval is exact when the population distribution is Normal and is approximately correct for large n in other cases.

Example 5: A manufacturer of high-resolution video terminals must control the tension on the mesh of fine wires that lies behind the surface of the viewing screen. Too much tension will tear the mesh and too little will allow wrinkles. The tension is measured by an electrical device with output readings in millivolts (mV). Some variation is inherent in the production process. Careful study has shown that when the process is operating properly, the standard deviation of the tension readings is σ = 43 mV. Here are the tension readings from an SRS of 20 screens from a single day's production:

269.5	297.0	269.6	283.3	304.8	280.4	233.5	257.4	317.5	327.4
264.7	307.7	310.0	343.3	328.1	342.6	338.8	340.1	374.6	336.1

Construct and interpret a 90% confidence interval for the mean tension μ of all the screens produced on this day.

The population is all screens ~~produced~~ that day.

The parameter is μ, the average ~~mean~~ tension

- The sample is an SRS.
- Screens are independent since it is reasonable to say there are more than 200 screens produced per day.
- Shape of population is unknown and sample size is too small so we check boxplot. The boxplot appears symmetric, so there is no reason to suspect Normality. We assume the distribution of $\bar{x}$ to be approx Normal.

$\bar{x} = 306.32 \quad z^* = 1.645$

$306.32 \pm 1.645\left(\frac{43}{\sqrt{20}}\right)$

$290.50 - 322.14$

We are 90% confident that the true average tension of the screens is between 290.5 - 322.14 mV.

Example 6: Questionnaires were sent to 180 teachers in the Philadelphia area and 155 were returned. The average number of years of teaching of these 155 teachers was 13.6 years. Assume that the population standard deviation of years teaching is 7.3 years. Find

The population is the teachers in the Philadelphia area. The parameter is μ, the average years of teaching.

We have to assume its an SRS so procede with caution

...ay affect
...x ability
...o generalize
...r results

Since the sample size is large, $\bar{x}$ is approx Normally distributed.

We have to assume there are more than 1800 teachers in Philadelphia which is probably reasonable

a) a 95% confidence interval $\bar{X} = 13.6$ $\sigma = 7.3$ $n = 155$ $z^* = 1.96$

(12.45, 14.75)

$13.6 \pm 1.96\left(\frac{7.3}{\sqrt{155}}\right)$

b) a 90% confidence interval $z^* = 1.645$

$13.6 \pm 1.645\left(\frac{7.3}{\sqrt{155}}\right)$

(12.64 - 14.56)

c) a 99% confidence interval $z^* = 2.576$

$13.6 \pm 2.576\left(\frac{7.3}{\sqrt{155}}\right)$

(12.09, 15.11)

d) Explain in words what your 95% confidence interval says in the context of the problem

We are 95% Confident the true average number of years taught in the Philadelphia area is between 12.45 years and 14.75 years

Example 7: The number of French fries in a Medium container of fries at McDonald's is not always the same. Here is the number of fries for a sample of 30 orders in different Bucks County McDonalds.

84	75	82	91	95	77	78	77	82	80
80	84	85	79	83	85	88	69	84	81
86	86	85	88	84	91	83	80	78	82

a. We expect the distribution of fries to be normal. Make a stemplot or histogram of these 30 values and describe its shape.

boxplot looks fair symmetrical

Histogram - somewhat bell shaped

b. Suppose that McDonalds published the standard deviation σ of the number of fries in a medium container to be 4.2 fries. Find a 98% confidence interval for the number of fries.

The population is all medium orders of fries served in the county. The parameter is the avg # of fries in one order. We have to assume it is an SRS so procede with caution. Orders are independent since its reasonable to believe there are at least 300 orders served. We can assume $\bar{x}$ is approx Normal since the sample size is large + graph gives us no reason to suspect.

$82.7 \pm 2.326(4.2/\sqrt{30})$ (80.9, 84.5)

c. Explain in words what b) above means in the context of the problem.

We are 98% confident the true average # of fries in a Medium container in Bucks County to be between 82.7 and 84.5 fries.

d. Would you trust the conclusion in part b if all the fries came from one McDonalds? Why?

NO this is definitely not an SRS so we can't generalize our results to all McDonalds in the county.

Inference Toolbox:

To construct a confidence Interval:

1. Identify the population of interest and the parameter to be estimated

2. Check the conditions (SRS, Normality, Independence)

3. Calculations : CI = estimate ± margin of Error

4. Interpretation

How confidence intervals behave

Typically a person performing an observational study chooses the confidence he desires and the margin of error follows from this choice. We usually want high confidence and a small margin of error, but we cannot have both. There is usually a trade-off.

- If we ask for high confidence, we have to allow ourselves a large margin of error.

Example 8: If I want to predict your average in a course with 99% confidence, I might say that I am 99% confident that you will get a 75% with a margin of error of 25%. That is saying that we are 99% confident that you will get between 50% and 100%. Notice that this doesn't say much other than you will probably pass the course.

- If we want a small margin of error, we have to ask for a smaller confidence level.

Example 9: If I want to predict your average with a margin of error of 2 points, I might say that I am 50% confident that you will get a 92% with a margin of error of 2 percentage points. That is saying that I am 50% confident that you will get between a 90 and 94 in the course. Again, the small range is impressive but with 50% confidence, I am not very confident at all. It is a coin flip.

Ways to make the margin of error smaller:

1. Z^* gets smaller but a smaller a* means a shorter CI so lower confidence!

2. σ gets smaller (difficult to do in the real world!)

3. n gets larger!

   Reducing the variability will create a lower margin of error

   Remember that quadrupling the sample size will half the margin of error!

Example 10: Suppose that the manufacturer in example 5 wants 99% confidence rather than 90%. Construct this CI. What happened to the margin of error? Explain.

$$306.32 \pm 2.576\left(\frac{43}{\sqrt{20}}\right)$$

$$306.32 \pm 24.77$$

$$(281.55, 331.09)$$

moe is larger due to higher C

Determining the sample size

Sometimes we wish to establish a specified margin of error for a certain confidence level. That fixes z^* and σ certainly cannot change. The only way we can achieve what we want is to change n, the sample size.

> Sample size for desired Margin of Error:
>
> To determine the sample size n for a specified Margin of Error:
>
> $z^* \sigma/\sqrt{n} \leq$ Margin of Error
>
> $z^* \frac{\sigma}{\sqrt{n}} \leq$ M.o.E

Example 11: In the French fry problem above, how many samples would we need to take in order to have the margin of error be less than 0.75 fries with 95% confidence?

$\sigma = 4.2$ $z^* = 1.96$

$1.96\left(\frac{4.2}{\sqrt{n}}\right) \leq .75$

$8.232 \leq .75\sqrt{n}$

$10.976 \leq \sqrt{n}$

$120.47 \leq n$

121 orders

Example 12: The standard deviation of purchases made at a Wawa store is \$2.11. We wish to predict the average price of a Wawa sale with 98% confidence with a margin of error of less than 25 cents. How many purchases must we sample? If we find that we can accept 90% confidence, then how many should we sample?

$z^* = 2.326$

$2.326\left(\frac{2.11}{\sqrt{n}}\right) \leq .25$

$19.63 \leq \sqrt{n}$

385.39

386 purchases

$z^* = 1.645$

$1.645\left(\frac{2.11}{\sqrt{n}}\right) \leq .25$

$192.76 \leq n$

193 purchases

NOTE: Taking observations costs time and money. The required sample size may be impossibly expensive. **Notice that it is the size of the sample that determines the margin of error. The size of the population does not influence the sample size we need.** (This is true as long as the population is much larger than the sample.)

Finally, some warnings:

Our handy-dandy formula for producing confidence intervals: $\bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$ is fraught with dangers:

- The data must come from an SRS.
- The margin of error in a confidence interval is based only on randomness and not because of issues of under-coverage and non response.
- The formula is not accurate for probability samples more complex than an SRS. Stratified and multistage samples have their own set of formulas and are not covered in this course.
- If there is any bias in your sample, this and any method is useless. Garbage in-garbage out.
- Because $\bar{x}$ is strongly influence by extreme observations, outliers can have a large effect on the confidence interval (as we saw in homework problem 8). Outliers should be removed if possible. There are procedures to deal with outliers to generate confidence intervals but again, they are not covered in this course.
- If the sample size n is small and the population is not normal, the formula will not be accurate. It is best to have a sample size $n \geq 15$ in these procedures, especially if your sample shows strong skew.
- Again, you must know the standard deviation σ of the population. This is unrealistic and thus we rarely use this exact procedure in real-life. We are only doing it to understand the procedure so when we get to a later chapter and are faced with problems where we do not know σ, you will understand the process.

Example 13: A talk show opinion poll A radio talk show invites listeners to enter a dispute about a proposed pay increase for city council members. "What yearly pay do you think council members should get? Call us with your number." In all, 958 people call. The mean pay they suggest is $\bar{x}$ = \$8740 per year, and the standard deviation of the responses is s = \$1125. For a large sample such as this, s is very close to the unknown population σ. The station calculates the 95% confidence interval for the mean pay μ that all citizens would propose for council members to be \$8669 to \$8811.

(a) Is the station's calculation correct? ~~No~~E

yes just some rounding

$1.96\left(\frac{1125}{\sqrt{958}}\right) = 71.24$

$(8668.76, 8811.24)$

(b) Does their conclusion describe the population of all the city's citizens? Explain your answer.

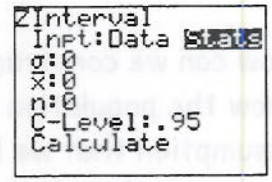
NO - Not an SRS!

Can't generalize

Voluntary response!

Using the Calculator and CI

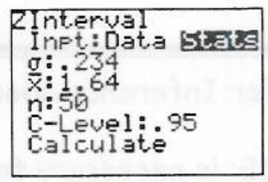
The TI-84 has built-in tests for inferences (which is what taking confidence intervals is). They are found in the $\boxed{\text{STAT}}\boxed{\text{TESTS}}$ menu. We want #7:ZInterval…. Which gives a screen similar to the one to the right:

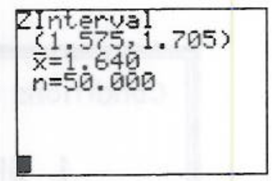


Let's do # 3 from the examples above using the calculator:

The price of a two-liter bottle of Pepsi nationwide has a standard deviation σ of 23.4 cents. A random sample of 50 bottles found an average price of \$1.64.

In this case, we are given summary statistics, not the actual data itself. So In the ZInterval menu, you use Stats. There are 4 inputs, $\sigma, \bar{x}, n$, and C - Level, and we know all of them. Simply input them, remembering to put the confidence level in decimal form: Then press Calculate.





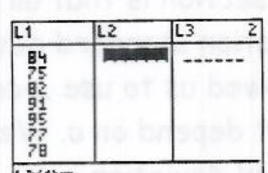
The calculator gives you the interval in up to 4-decimal place accuracy. Your answers might not completely match the ones done by formula because in those, your values of z* was not completely accurate. Thus the calculator's answers will always be more accurate than the ones done by formula and it is recommended that you use the calculator. However, it is vital that you understand the process of confidence intervals as you will be tested on it in the AP exam. Also note that although the calculator doesn't directly give the margin of error, you could easily find it by either subtracting $\bar{x}$ from the upper bound or subtracting the lower bound from $\bar{x}$.

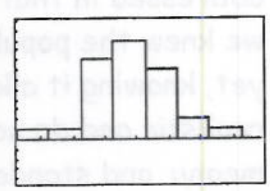
If you are given raw data, the calculator can find confidence intervals on those as well. Let's repeat problem # 5 above.

The number of French fries in a Medium container of fries at McDonald's is not always the same. Here are the number of fries for a sample of 30 orders in different Bucks County McDonalds. Suppose that McDonalds published the standard deviation σ of the number of fries in a medium container to be 4.2 fries. Find a 98% confidence interval for the number of fries

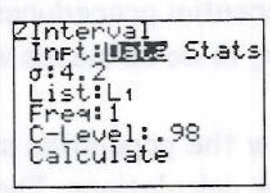
84	75	82	91	95	77	78	77	82	80
80	84	85	79	83	85	88	69	84	81
86	86	85	88	84	91	83	80	78	82

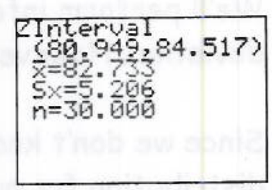
When we did this problem, we had to calculate our value of $\bar{x}$ and it was prudent to examine the data for outliers. We can, of course, do this by inputting our data into a list and generating a histogram.





When we choose ZInterval from our $\boxed{\text{STAT}}\boxed{\text{TESTS}}$ menu, we now choose **Data**. You still input σ, your choice of Lists, and the confidence level. The upper and lower bound are given along with $\bar{x}$ and Sx, the sample standard deviation. Again, you can easily find the margin of error by either subtracting $\bar{x}$ from the upper bound or subtracting the lower bound from $\bar{x}$.





It is strongly recommended that you redo homework problems using the calculator.